CHAPTER 9

Probability

"What do games have to do with math?"

Rolling number cubes allows you to advance forward or backward in many board games. **Rolling doubles on a pair of number cubes happens only about 17% of the time.** In mathematics, you can use probability to help you determine how likely it is for events to take place.

You will solve problems about games in Lesson 9-1.

KS Studios

CONTENTS

GETTING STARTED

Take this quiz to see whether you are ready to begin Chapter 9. Refer to the lesson or page number in parentheses if you need more review.

Vocabulary Review

Choose the correct term or number to complete each sentence.

1. A fraction is in (factored, simplest) form when the GCF of the numerator and denominator is 1. (Lesson 5-3)
2. The fraction $\frac{12}{38}$ in simplest form is $\left(\frac{6}{19}, \frac{3}{17}\right)$. (Lesson 5-3)

Prerequisite Skills

Multiply.

3. 7×15
4. 24×6
5. 13×4
6. 8×21
7. 5×32
8. 30×8
9. $6 \cdot 5 \cdot 4 \cdot 3$
10. $7 \cdot 6 \cdot 5$
11. $8 \cdot 7 \cdot 6$
12. $4 \cdot 3 \cdot 2 \cdot 1$
13. $10 \cdot 9 \cdot 8 \cdot 7$
14. $11 \cdot 10 \cdot 9$

Write each fraction in simplest form. Write *simplified* if the fraction is already in simplest form. (Lesson 5-3)

15. $\frac{8}{12}$
16. $\frac{3}{18}$
17. $\frac{4}{9}$
18. $\frac{5}{15}$

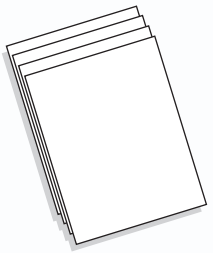
Find each value. (Lesson 5-3)

19. $\frac{6 \cdot 5}{3 \cdot 2}$
20. $\frac{9 \cdot 8 \cdot 7}{5 \cdot 4 \cdot 3}$
21. $\frac{4 \cdot 3 \cdot 2}{3 \cdot 2 \cdot 1}$
22. $\frac{7 \cdot 6 \cdot 5 \cdot 4}{4 \cdot 3 \cdot 2 \cdot 1}$

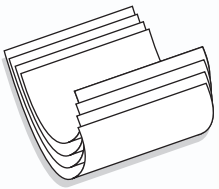
FOLDABLES Study Organizer

Probability Make this Foldable to help you organize the topics in this chapter. Begin with four sheets of $8\frac{1}{2}''$ by 11″ paper.

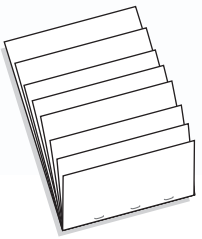
STEP 1 **Stack Pages**
Place 4 sheets of paper $\frac{3}{4}$ inch apart.



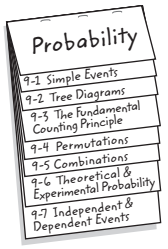
STEP 2 **Roll Up Bottom Edges**
All tabs should be the same size.



STEP 3 **Crease and Staple**
Staple along fold.



STEP 4 **Label**
Write the chapter title on the front. Label each tab with a lesson number and title.



Reading and Writing As you read and study each lesson, write notes and examples under the corresponding tab.

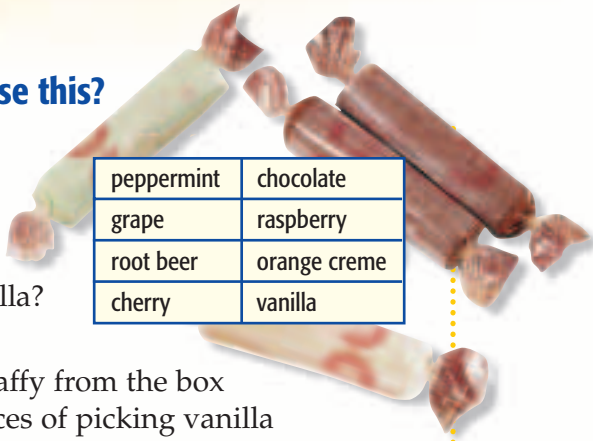
9-1 Simple Events

Academic Standards
7.6.5

What You'll Learn
Find the probability of a simple event.

NEW Vocabulary
- outcome
- simple event
- probability
- random
- complementary event

WHEN am I ever going to use this?



CANDY A box of saltwater taffy contains 48 pieces, six pieces of each flavor shown at the right.

peppermint	chocolate
grape	raspberry
root beer	orange creme
cherry	vanilla

1. What fraction of the taffy is vanilla? Write in simplest form.
2. Suppose you take one piece of taffy from the box without looking. Are your chances of picking vanilla the same as picking root beer? Explain.

The 48 pieces of taffy in the box above are called **outcomes**. A **simple event** is one outcome or a collection of outcomes. For example, picking a piece of vanilla taffy is a simple event. The chance of that event happening is called **probability**.

Key Concept — **Probability**

Words The probability of an event is a ratio that compares the number of favorable outcomes to the number of possible outcomes.

Symbols $P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$

Outcomes occur at **random** if each outcome occurs by chance. A piece of vanilla taffy was selected at random in the activity above.

Read $P(\text{vanilla})$ as *the probability of picking a piece of vanilla taffy.*

$$P(\text{vanilla}) = \frac{6}{48} \quad \begin{matrix}\leftarrow \text{number of pieces of vanilla taffy} \\ \leftarrow \text{total number of pieces of taffy}\end{matrix}$$

$$= \frac{1}{8}, \text{ or } 12.5\% \quad \text{Simplify.}$$

EXAMPLE 1 **Find Probability**

What is the probability of rolling an even number on a number cube marked with 1, 2, 3, 4, 5, and 6 on its faces?

$$P(\text{even number}) = \frac{\text{even numbers possible}}{\text{total numbers possible}}$$

$$= \frac{3}{6} \quad \text{Three numbers are even: 2, 4, and 6.}$$

$$= \frac{1}{2} \quad \text{Simplify.}$$

The probability of rolling an even number is $\frac{1}{2}$ or 50%.

EXAMPLE 2 Find Probability

PARCHEESI **Jewel rolls two number cubes. She can move her game piece if a total of 5 is shown on the number cubes, or if a 5 is shown on at least one number cube. What is the probability that she can move her game piece on one roll of the number cubes?**

List all the possible outcomes. Then, find the pairs that total 5, or if a 5 is shown on at least one of the number cubes.

1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

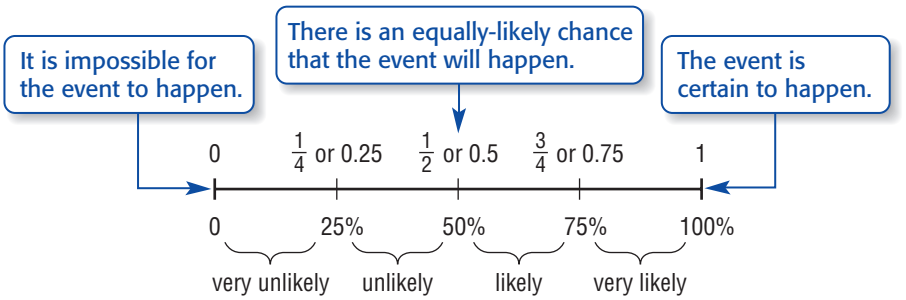
There are 36 possible outcomes and 15 of them are favorable. So, the probability that Jewel moves her game piece in one roll is $\frac{15}{36}$, or $\frac{5}{12}$.

REAL-LIFE MATH

PARCHEESI Parcheesi is derived from the Indian word *pacis*, which means 25.

Source: www.yesterdayland.com

The probability that an event will happen is somewhere between 0 and 1. It can be shown on a number line.



Either Jewel will be able to move her game piece or she will *not* be able to move it. Two events that are the only ones that can possibly happen are examples of **complementary events**.

EXAMPLE 3 Find a Complementary Event

PARCHEESI **Refer to Example 2. Find the probability that Jewel *cannot* move her game piece on one roll of the number cubes.**

$$P(A) + P(\text{not } A) = 1$$

$$\frac{5}{12} + P(\text{not } A) = 1 \quad \text{Substitute } \frac{5}{12} \text{ for } P(A).$$

$$-\frac{5}{12} \qquad -\frac{5}{12} \quad \text{Subtract } \frac{5}{12} \text{ from each side.}$$

$$P(\text{not } A) = \frac{7}{12} \quad \text{Simplify.}$$

So, the probability that Jewel *cannot* move her game piece is $\frac{7}{12}$.

STUDY TIP

Check Your Answer You can check the answer by counting the number of outcomes that do not total 5 and that do not have a 5 on at least one number cube.

Matt Meadows

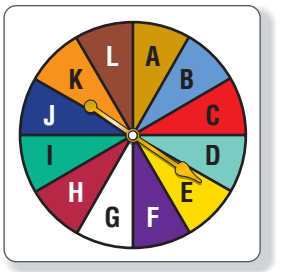
Skill and Concept Check

Writing Math — Exercises 1 & 3

1. **Explain** why an event with a probability of 0.7 is likely to happen.
2. **OPEN ENDED** Describe an event that has a probability of $\frac{1}{5}$.
3. **Which One Doesn't Belong?** Identify the pair of probabilities that do not represent probabilities of complementary events. Explain.

$\frac{5}{8}, \frac{3}{8}$	0.65, 0.55	$\frac{4}{6}, \frac{1}{3}$	0.875, $\frac{1}{8}$

GUIDED PRACTICE



Use the spinner at the right to find each probability. Write as a fraction in simplest form.

4. P(J)
5. P(vowel)

A bag contains 7 blue, 5 purple, 12 red, and 6 orange marbles. Find each probability if you draw one marble at random from the bag. Write as a fraction in simplest form.

6. P(purple)
7. P(red or orange)
8. P(not blue)
9. **CANDY** Refer to the activity on the top of page 370. What is the probability of picking a piece of taffy that is *not* peppermint nor root beer? Write the probability as a percent.

Practice and Applications

HOMEWORK HELP

For Exercises	See Examples
10–15	1
18, 20–23, 25–27	2
17, 19, 24, 28–29	3

Extra Practice
See pages 584, 604.

A set of 20 cards is numbered 1, 2, 3, …, 20. Suppose you pick a card at random without looking. Find the probability of each event. Write as a fraction in simplest form.

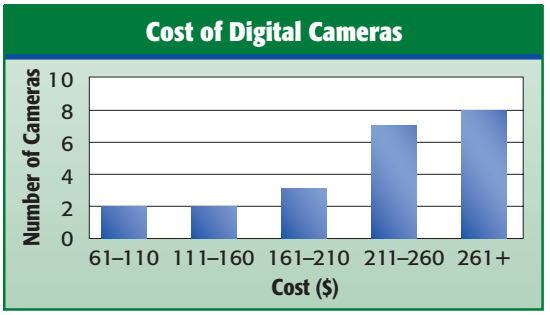
10. P(1)
11. P(not a factor of 10)
12. P(multiple of 3)
13. P(even number)
14. P(less than or equal to 20)
15. P(3 or 13)
16. How likely is it that an event with a probability of 0.28 will occur?
17. The forecast for tomorrow says that there is a 37% chance of rain. Describe the complementary event and its probability.

STUDENT COUNCIL **The table shows the members of the Student Council. Suppose one student is randomly selected as the president. Find the probability of each event. Write as a fraction in simplest form.**

Student Council	
girls	30
boys	20
8th graders	25
7th graders	15
6th graders	10

18. P(girl)
19. P(not 7th grader)
20. P(boy)
21. P(8th grader)
22. P(boy or girl)
23. P(6th or 8th grader)
24. P(not 6th grader)
25. P(5th grader)
26. Which event has a greater chance of happening: picking a president who is a girl or a president who is not an 8th grader? Explain.

27. **TECHNOLOGY** The graph shows the cost of 22 digital cameras. If one of the 22 cameras is chosen at random, what is the probability that it costs between $111 and $160?



28. **MOVIES** The probability of buying a defective DVD is 0.002. What is the probability of buying a DVD that is *not* defective?

29. **MULTI STEP** The Jefferson Middle School Booster Club is selling raffle tickets for a new computer system. They sold 1,000 tickets at $2 each. Emilia's parents spent $200 on tickets. What is the probability that they will *not* win?

30. **CRITICAL THINKING** Melissa and Hakan are playing a game by rolling two number cubes. Hakan gets a point each time the sum of the number cubes is 2, 3, 4, 9, 10, 11, or 12. Melissa gets a point when the sum is 5, 6, 7, or 8. Does each player have an equal chance to win? Explain.

Standardized Test Practice and Mixed Review

ISTEP+ Practice

31. **MULTIPLE CHOICE** A bookshelf contains the books at the right. If you randomly choose a book from the shelf, what is the probability that it is science fiction?

(A) $\frac{1}{10}$ (B) $\frac{3}{20}$ (C) $\frac{9}{40}$ (D) $\frac{1}{4}$

32. **MULTIPLE CHOICE** In a school raffle, one ticket will be drawn out of a total of 500 tickets. If the Hawkins family has 12 tickets, what is the probability that they will win?

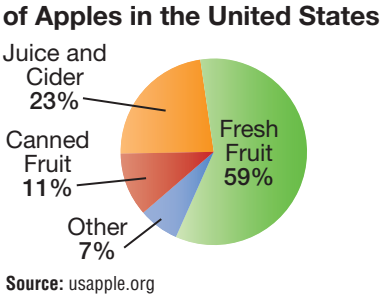
(F) 0.002 (G) 2% (H) $\frac{3}{125}$ (I) $0.08\overline{3}$

Find the interest earned to the nearest cent for each principal, interest rate, and time. (Lesson 8-6)

33. $300, 10%, 2 years

34. $900, 5.5%, 4.5 years

35. **FOOD** The United States produced almost 11 billion pounds of apples in 2000. Use the information in the graph to find how many pounds of apples were used to make juice and cider. (Lesson 7-8)



Write each fraction or mixed number as a decimal. Use bar notation if necessary. (Lesson 5-4)

36. $\frac{3}{5}$ 37. $\frac{9}{20}$ 38. $6\frac{1}{8}$ 39. $1\frac{7}{9}$

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL **Write each fraction in simplest form. Write *simplified* if the fraction is already in simplest form.** (Lesson 5-3)

40. $\frac{2}{6}$ 41. $\frac{3}{8}$ 42. $\frac{15}{30}$ 43. $\frac{6}{16}$ 44. $\frac{18}{32}$

9-2 Tree Diagrams

Academic Standards
7.6.7

What You'll Learn
Use tree diagrams to count outcomes and find probabilities.

NEW Vocabulary
fair game
tree diagram
sample space

HANDS-ON Mini Lab

Materials
- 2 counters
- marker

Work with a partner.

Here is a probability game that you can play with two counters.

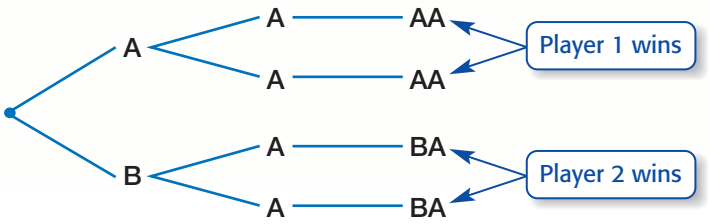
- Mark one side of the first counter A. Mark the other side B. Mark both sides of the second counter A.
- Player 1 tosses the counters. If both sides shown are the same, Player 1 wins a point. If the sides are different, Player 2 wins a point. Record your results.
- Player 2 then tosses the counters and the results are recorded. Continue alternating the tosses until each player has tossed the counters ten times. The player with the most points wins.

1. Before you play, make a conjecture. Do you think that each player has an equal chance of winning? Explain.
2. Now, play the game. Who won? What was the final score?
3. Collect the data from the entire class. What is the combined score for Player 1 versus Player 2?
4. Do you want to change the conjecture you made in Exercise 1? Explain.

A game in which players of equal skill have an equal chance of winning is a **fair game**. One way you can analyze whether games are fair is by drawing a **tree diagram**. A tree diagram is used to show all of the possible outcomes, or **sample space**, in a probability experiment.

EXAMPLE Draw a Tree Diagram

1 **GAMES** **Refer to the Mini Lab above. Draw a tree diagram to show the sample space. Then determine whether the game is fair.**



There are four equally-likely outcomes with two favoring each player. So, the probability that each player can win is $\frac{1}{2}$. Thus, the game is fair.

CONTENTS

EXAMPLE Find the Number of Outcomes

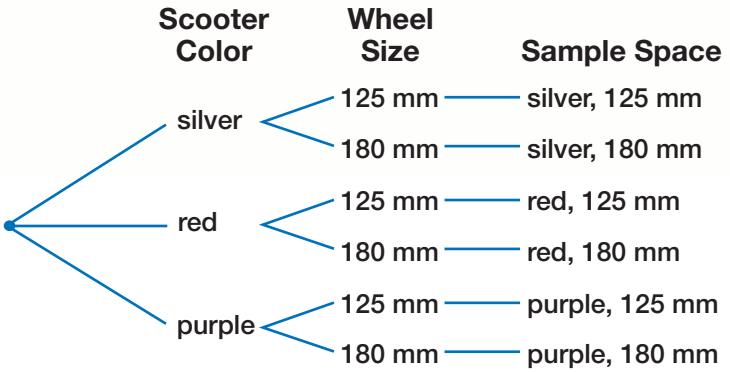
REAL-LIFE MATH

SCOOTERS Kickboard scooters can travel approximately 4 miles per hour.

Source: www.factmonster.com

2 **SCOOTERS** **A certain type of kickboard scooter comes in silver, red, or purple with wheel sizes of 125 millimeters or 180 millimeters. Find the total number of color-wheel size combinations.**

Make a tree diagram to show the sample space.



There are six different color and wheel size combinations.

Your Turn

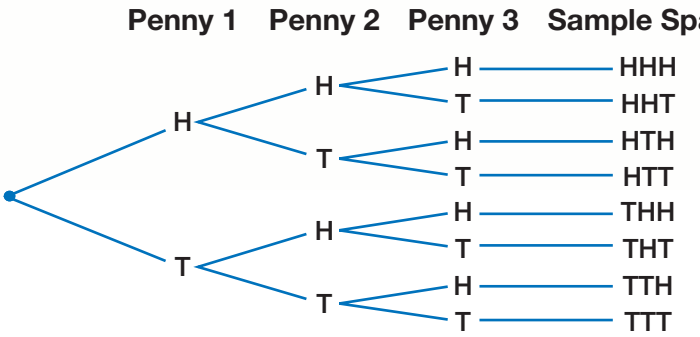
a. How many outcomes are there if the manufacturer added an orange color and a 150-millimeter wheel size?

You can also use tree diagrams to help you find the probability of events.

EXAMPLE Find Probability Using Tree Diagrams

3 **COINS** **Suppose Pablo tosses three pennies. Find the probability that all three will show heads.**

Make a tree diagram to show the sample space. Then, find the probability of the three pennies showing heads.



The sample space contains 8 possible outcomes. Only 1 outcome has all pennies showing heads. So, the probability of three pennies showing heads is $\frac{1}{8}$, or 12.5%.

Your Turn **Find each probability.**

b. P(3 tails) **c.** P(exactly 2 heads) **d.** P(at least 1 tail)

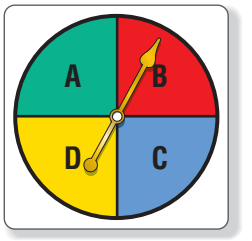
Skill and Concept Check

Writing Math
Exercises 1 & 2

1. **Describe** a fair game between two players using one coin.
2. **OPEN ENDED** Give an example of a situation that has 8 outcomes.

GUIDED PRACTICE

The spinner is spun twice.



3. Draw a tree diagram to represent the situation.
4. How many outcomes are possible?
5. What is the probability of spinning two Cs?
6. **DELICATESSEN** A neighborhood deli sells sandwiches that can be made with ham, turkey, roast beef, salami, or bologna on rye, white, pumpernickel, or sourdough breads. Make a tree diagram to show all of the possible meat-bread choices.

Practice and Applications

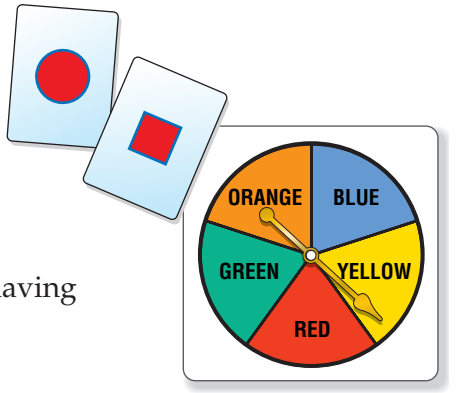
HOMEWORK HELP

For Exercises	See Examples
7–15, 18, 21	1, 2
16–17, 19	3

Extra Practice
See pages 585, 604.

For each situation, make a tree diagram to show the sample space. Then give the total number of outcomes.

7. tossing a coin and rolling a number cube
8. choosing black, blue, or brown socks with boots, gym shoes, or dress shoes
9. picking a number from 1 to 5 and choosing the color red, white, or blue
10. choosing a card with a shape and spinning the spinner from the choices at the right
11. choosing a letter from the word SPACE and choosing a consonant from the word MATH
12. choosing a purple, green, black, or silver mountain bike having 10, 18, 21, or 24 speeds
13. tossing a quarter, a dime, and a nickel
14. rolling a number cube, tossing a coin, and choosing a card from the cards marked A and B



FAMILY **For Exercises 15–17, use the information below.**
Mr. and Mrs. Chen have three children. Suppose the chance of having either a boy or a girl is 50%.

15. Draw a tree diagram showing the possible arrangements of gender of the three children.
16. What is the probability of having 2 boys and 1 girl?
17. What is the probability of having the first two children boys and the last child a girl?

UNIFORMS **For Exercises 18 and 19, use the information below and at the right.**

Hunter is a big fan of the Houston Astros baseball team and wears a different jersey and cap every time he goes to a game. The table shows the number of different jerseys and caps Hunter owns.

	Home Jersey	Road Jersey	Practice Jersey	Cap
White	2	0	2	0
Black	1	0	0	1
Gray	0	1	0	0
Orange	1	1	0	2

18. How many jersey/cap combinations can Hunter wear when he goes to the baseball game to cheer on the Astros?

19. If Hunter picks a jersey/cap combination at random, what is the probability that he will wear a home jersey with a black cap?

20. **RESEARCH** Use the Internet or another resource to find the number of jerseys and caps your favorite major league baseball team has as part of its uniform. How many jersey/cap combinations are there for the team you picked?

21. **GAMES** Determine whether the following game for two players is fair. Explain.

- Three counters are labeled according to the table at the right.
- Toss the three counters.
- If exactly 2 counters match, Player 1 scores a point. Otherwise, Player 2 scores a point.

	Side 1	Side 2
Counter 1	A	B
Counter 2	A	C
Counter 3	B	C

22. **CRITICAL THINKING** Refer to Exercise 21. Adjust the scoring of the game so that it is fair.

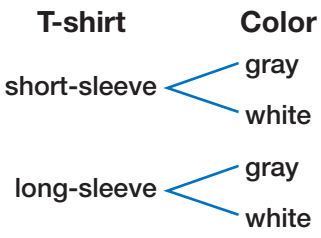
Standardized Test Practice and Mixed Review

ISTEP+ Practice

23. **MULTIPLE CHOICE** How many three-course dinners are possible if Yolanda chooses from an appetizer of soup or salad; an entree of steak, chicken, or fish; and a dessert of cake or pie?

(A) 7 (B) 10 (C) 12 (D) 15

24. **SHORT RESPONSE** Find the sample space of the tree diagram.



PROBABILITY **A spinner is equally likely to stop on each of its regions numbered 1 to 20. Find each probability as a fraction in simplest form.** (Lesson 9-1)

25. a prime number 26. GCF(12, 18) 27. multiple of 2 or 3

28. **INTEREST** If Carlota invests $2,100 in a CD for 5 years at a simple interest rate of 4.75%, how much will the CD be worth after 5 years? (Lesson 8-7)

GETTING READY FOR THE NEXT LESSON

BASIC SKILL **Multiply.**

29. $7 \cdot 22$ 30. $5 \cdot 36$ 31. $11 \cdot 16$ 32. $23 \cdot 20$ 33. $131 \cdot 4$

9-3 The Fundamental Counting Principle

What You'll Learn
Use multiplication to count outcomes.

NEW Vocabulary
Fundamental Counting Principle

WHEN am I ever going to use this?

RETAIL SALES The Jean Factory sells juniors' jeans in different sizes and lengths. The table shows what they have available.

Sizes	Lengths
3	petite
5	regular
7	tall
9	
11	
13	
15	

1. According to the table, how many sizes of juniors' jeans are there?
2. How many lengths are there?
3. Find the product of the two numbers you found in Exercises 1 and 2.
4. Draw a tree diagram to help you find the number of different size and length combinations. How does the number of outcomes compare to the product you found above?

In the activity above, you discovered that multiplication, instead of a tree diagram, can be used to find the number of possible outcomes in a sample space. This is called the **Fundamental Counting Principle**.

Key Concept **Fundamental Counting Principle**

If event M can occur in m ways and is followed by event N that can occur in n ways, then the event M followed by N can occur in $m \times n$ ways.

EXAMPLE 1 Use the Fundamental Counting Principle

FOOD **A famous steak house allows customers to create their own steak dinners. The choices are shown at the right. How many different steak dinners are possible?**

451 Steakhouse

Steak	How Steaks Are Cooked	Potatoes
New York Strip	rare	mashed
Ribeye	medium	baked
Filet	well	twice baked
Porterhouse		au gratin
T-Bone		

$$\underbrace{5}_{\text{types of steaks}} \cdot \underbrace{3}_{\text{number of ways steaks can be cooked}} \cdot \underbrace{4}_{\text{types of potatoes}} = \underbrace{60}_{\text{total number of steak dinners}}$$

There are 60 different ways of choosing a steak dinner.

Check You can check your work by drawing a tree diagram and listing the 60 outcomes.

Geoff Butler

CONTENTS

ISTEP+ Practice

Standardized Test Practice

EXAMPLE 2 Find Outcomes

MULTIPLE-CHOICE TEST ITEM There are 2 roads connecting Eastland and Harping, 3 roads connecting Harping and Sinclair, and 2 roads connecting Sinclair and Johnstown. Find the number of ways to drive from Eastland to Johnstown.

(A) 8 (B) 12 (C) 16 (D) 20

Read the Test Item

To find the number of ways to drive from Eastland to Johnstown, multiply the number of roads from Eastland to Harping, from Harping to Sinclair, and from Sinclair to Johnstown.

Solve the Test Item

There are 2 roads from Eastland to Harping, 3 roads from Harping to Sinclair, and 2 roads from Sinclair to Johnstown. So, there are $2 \cdot 3 \cdot 2$, or 12 ways to drive from Eastland to Johnstown. So the answer is B.

Test-Taking Tip (The Princeton Review)

Educated Guess Find out if there is a penalty for incorrect answers. If there is no penalty, making an educated guess can only increase your score, or at worst, leave your score the same.

Skill and Concept Check

Writing Math — Exercises 1–4

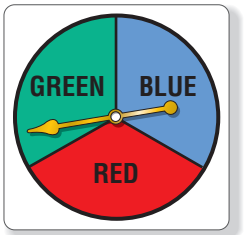
1. **Explain** how to use the Fundamental Counting Principle to count outcomes.
2. **OPEN ENDED** Give an example of a situation that has 48 outcomes using three events.
3. **NUMBER SENSE** Can a pair of events have a number of outcomes that is prime? Explain.
4. **Which One Doesn't Belong?** Identify the pair of events that do not have the same number of outcomes as the other three. Explain your reasoning.

12 meats, 8 breads	16 T-shirts, 6 colors
18 hats, 8 sizes	24 teams, 4 sports

GUIDED PRACTICE

Use the Fundamental Counting Principle to find the total number of outcomes in each situation.

5. tossing a quarter, a dime, and a nickel
6. choosing scrambled, sunny-side up, or poached eggs with bacon or sausage and milk, orange juice, or apple juice
7. choosing a card from twenty different cards and spinning the spinner at the right



8. **STUDENT GOVERNMENT** If Opal, Jacob, Luanda, Tyrone, and Erica have an equal chance of being an officer of student council, what is the probability that Luanda will be an officer?

Practice and Applications

HOMEWORK HELP

For Exercises	See Examples
9–14	1
15–16	2

Extra Practice
See pages 585, 604.

Use the Fundamental Counting Principle to find the total number of outcomes in each situation.

9. rolling a number cube and tossing two coins
10. choosing a number from 1 to 20 and a color from 7 colors
11. choosing a plain, blueberry, garlic, or cinnamon-and-raisin bagel, with plain, chive, or sun-dried tomato cream cheese
12. choosing iced tea in regular, raspberry, lemon, or peach flavors; sweetened or unsweetened; and in a glass or in a plastic container
13. picking a date in the month of May and a day of the week
14. choosing a 4-digit Personal Identification Number (PIN) if the digits cannot be repeated

15. **ADVERTISING** The Wake-Up Restaurant advertises that you can have a different pancake breakfast every day of the year. It offers 25 different kinds of pancakes and 14 flavored syrups. If the restaurant is open every day of the year, is its claim valid? Explain.

16. **GAMES** A player rolls five number cubes to score the maximum number of points. Find the number of outcomes possible in one roll.

CRITICAL THINKING How many outcomes are possible if you toss each of the following?

17. one coin
18. two coins
19. three coins
20. n coins

Standardized Test Practice and Mixed Review

ISTEP+ Practice

21. **MULTIPLE CHOICE** Elizabeth has 3 sweaters, 5 blouses, and 6 skirts that coordinate. How many different outfits can Elizabeth make?

(A) 105 (B) 90 (C) 45 (D) 30

22. **GRID IN** WritePen makes 8 different styles of pens in several colors with 2 types of grips. If the company makes 112 kinds of pens, how many different colors do they make?

PROBABILITY **What is the probability that the spinner shown at the right will stop on each of the following numbers? Write as a fraction in simplest form.** (Lesson 9-1)

23. an even number
24. a multiple of 4
25. a number less than 10

26. **SCHOOL** Gustavo must choose from two geography classes, three history classes, and two statistics classes for next year. Make a tree diagram to show all of the possible schedules he can arrange. (Lesson 9-2)

GETTING READY FOR THE NEXT LESSON

BASIC SKILL **Multiply.**

27. $3 \cdot 2 \cdot 1$
28. $9 \cdot 8 \cdot 7$
29. $5 \cdot 4 \cdot 3 \cdot 2$
30. $7 \cdot 6 \cdot 5 \cdot 4$

9-4 Permutations

What You'll Learn
Find the number of permutations of a set of objects.

NEW Vocabulary
permutation
factorial

MATH Symbols
4! four factorial

HANDS-ON Mini Lab

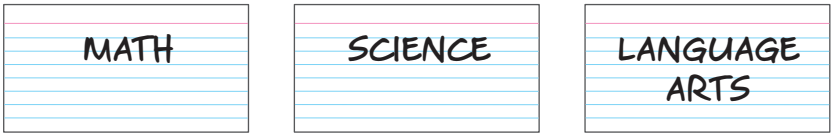
Materials
- 3 index cards

Work with a partner.

How many different ways are there to arrange your first three classes if they are math, science, and language arts?

STEP 1 Write math, science, and language arts on the index cards.

STEP 2 Choose one of the subjects as the first class of the day. Choose one of the remaining two subjects for the second class. The third class is the card that remains.



STEP 3 Record this arrangement of classes.

STEP 4 Change the order of the classes. Record this arrangement. Continue rearranging the cards until you have found all of the possible arrangements.

1. When you first started to make your list, how many choices did you have for your first class?
2. Once your first class was selected, how many choices did you have for the second class? Then, the third class?
3. Explain how you can use the Fundamental Counting Principle to find the number of arrangements.

A **permutation** is an arrangement, or listing, of objects in which order is important. You can use the Fundamental Counting Principle to find the number of possible arrangements.

There are **3** choices for the 1st class.
There are **2** choices that remain for the 2nd class.
There is **1** choice that remains for the 3rd class.

$3 \cdot 2 \cdot 1 = 6$ ← The number of arrangements of 3 classes.

The expression $3 \cdot 2 \cdot 1$ can be written as 3!, which is read *three factorial.*

Key Concept **Factorial**

The expression n **factorial** ($n!$) is the product of all counting numbers beginning with n and counting backward to 1.

EXAMPLES Evaluate Factorials

Find the value of each expression.

1 **7!**

$7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ Definition of factorial

$= 5{,}040$ Simplify.

2 **$2! \cdot 3!$**

$2! \cdot 3! = 2 \cdot 1 \cdot 3 \cdot 2 \cdot 1$ Definition of factorial

$= 12$ Simplify.

STUDY TIP

Technology You can find the factorial of a number on a calculator. To find 3!, press 3 PRB ▶ ▶ ENTER ENTER.

EXAMPLE Find a Permutation

3 **VOLLEYBALL** **In how many ways can the starting six players of a volleyball team stand in a row for a picture?**

This is a permutation that can be written as 6!.

$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ Definition of factorial

$= 720$ Simplify.

So, there are 720 ways the six starting players can stand in a row.

Arrangements can include all or only part of a group.

EXAMPLE Find a Permutation

4 **SWIMMING** **The finals of the Middle School Appalachian League features 8 swimmers. In how many ways can the swimmers finish in first or second place?**

There are 8 choices for first place and 7 choices that remain for second place. So, there are $8 \cdot 7$ or 56 choices for first and second place.

REAL-LIFE MATH

SWIMMING In 1996, swimmer Amy Van Dyken was the first American woman to win 4 gold medals at one Olympic Games.

Source: sportsillustrated.cnn.com

DUOMO/CORBIS

Skill and Concept Check

Writing Math Exercise 1

1. **Explain** the difference between 5! and $5 \cdot 4 \cdot 3$.
2. **OPEN ENDED** Describe a real-life situation that has 6 permutations.

GUIDED PRACTICE

Find the value of each expression.

3. 3!
4. $6! \cdot 2!$
5. In how many ways can you arrange the letters in the word *equal*?
6. **TRANSPORTATION** There are 7 students waiting at the bus stop. In how many ways can the students board the bus when it arrives?

Practice and Applications

HOMEWORK HELP

For Exercises	See Examples
7–14	1, 2
16	3
15, 17–18	4

Extra Practice
See pages 585, 604.

Find the value of each expression.

7. 5! **8.** 9! **9.** 4! · 3! **10.** four factorial

11. 3! · 6! **12.** 10 · 9 · 8 **13.** 5! · 4! **14.** 8! · 2!

15. In how many ways can a softball manager arrange the first four batters in a lineup of nine players?

16. How many different 5-digit zip codes are there if no digit is repeated?

17. MUSIC The chromatic scale has 12 notes. In how many ways can a song start with 4 different notes from that scale?

DOGS **For Exercises 18 and 19, use the information below and at the right.**
During the annual Westminster Dog Show, the best dog in each breed competes to win one of four top ribbons in the group.

18. In how many ways can a ribbon be awarded to a breed of dog in the Working group?

19. The top dog in each group competes against the other six group winners for Best of Show. If each dog has an equally-likely chance of winning Best of Show, what is the probability that a terrier will win?

2002 Westminster Dog Show

Group	Number of Breeds
Herding	19
Hounds	25
Non-sporting	18
Sporting	27
Terriers	27
Toy	22
Working	21

Source: westminsterkennelclub.org

Data Update Which breed of dog has won the most Best of Shows? Visit msmath2.net/data_update to learn more.

20. CRITICAL THINKING There are 1,320 ways for three students to win first, second, and third place during a debate match. How many students are there on the debate team?

Standardized Test Practice and Mixed Review

ISTEP+ Practice

21. SHORT RESPONSE In a Battle of the Bands contest, how many ways can the four participating bands be ordered?

22. GRID IN How many different three-digit security codes can be made from the digits 1, 2, 3, 4, and 5 if no digit is repeated in a code?

23. BREAKFAST Find the total number of outcomes if you can choose from 8 kinds of muffins, 3 sizes, and 4 beverages. (Lesson 9-3)

24. LUNCH Make a tree diagram showing different ways to make a sandwich with turkey, ham, or salami and either cheddar or Swiss cheese. (Lesson 9-2)

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL **Find each value.** (Lesson 5-3)

25. $\frac{5 \cdot 4}{2 \cdot 1}$ **26.** $\frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1}$ **27.** $\frac{5 \cdot 4 \cdot 3}{4 \cdot 3 \cdot 2}$ **28.** $\frac{10 \cdot 9 \cdot 8 \cdot 7}{8 \cdot 7 \cdot 6 \cdot 5}$

CHAPTER 9

Mid-Chapter Practice Test

Vocabulary and Concepts

1. **Explain** what it means for an event to occur at random. (Lesson 9-1)
2. **State** the Fundamental Counting Principle. (Lesson 9-3)
3. **Define** *permutation*. (Lesson 9-4)

Skills and Applications

Two number cubes are rolled. Find each probability. (Lesson 9-1)

4. P(sum of 6)
5. P(sum less than 7)
6. P(sum of 7 or 11)

For each situation, make a tree diagram to show the sample space. Then give the total number of outcomes. (Lesson 9-2)

7. tossing a penny and tossing a dime
8. choosing cereal, French toast, or pancakes and choosing orange, apple, cranberry or grapefruit juice

Use the Fundamental Counting Principle to find the total number of outcomes in each situation. (Lesson 9-3)

9. choosing a dinner with one entrée, one salad, and one dessert
10. rolling a number cube and tossing three coins

Entrée	Salad	Dessert
ham	potato	ice cream
beef	tossed	pie
turkey	cole slaw	cake

Find the value of each expression. (Lesson 9-4)

11. $5!$
12. $8!$
13. $2! \cdot 7!$

14. **STUDENT COUNCIL** In how many ways can a president, treasurer, and a secretary be chosen from among 8 candidates? (Lesson 9-4)

Standardized Test Practice

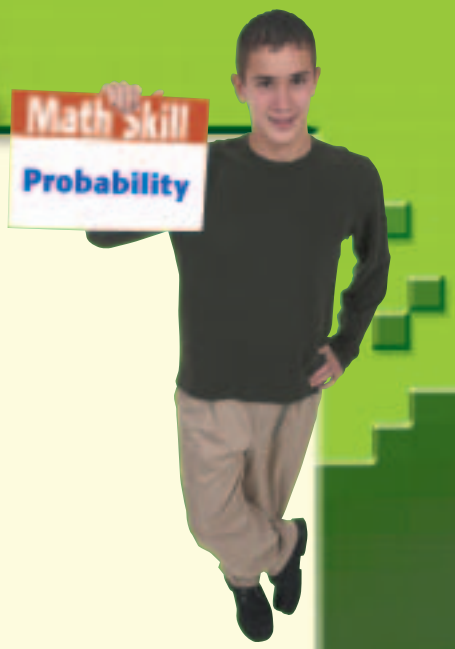
ISTEP+ Practice

15. **GRID IN** Jeffrey has 2 action, 3 comedy, and 4 drama DVDs. If he randomly picks one DVD to watch, what is the probability that it will be a comedy? (Lesson 9-1)

16. **SHORT RESPONSE** Molly has three windows to display three of five best-selling books. How many different displays can she make if she puts one book in each window? (Lesson 9-4)

The Game Zone

A Place To Practice Your Math Skills



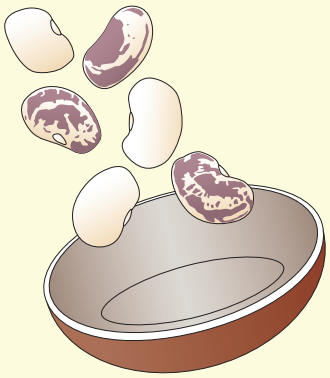
Cherokee Butterbean Game

GET READY!

Players: two, three, or four
Materials: 6 dry lima beans, bowl, marker

GET SET!

- This is a variation of a traditional Cherokee game.
- Color one side of each bean with the marker.



GO!

- The first player places the beans in the bowl, gently tosses the beans into the air, and catches them in the bowl. Points are scored as follows.
  - If all of the beans land with the marked or unmarked sides up, score 6 points.
  - If exactly one bean lands with the marked or unmarked side up, score 3 points.
  - If three beans are marked and three beans are unmarked, score 1 point.
- If a toss scores points, the player takes another turn. If a toss does not score any points, it is the next player's turn.
- **Who Wins?** The person with the most points after a given number of rounds is the winner.

9-5a HANDS-ON LAB

A Preview of Lesson 9-5

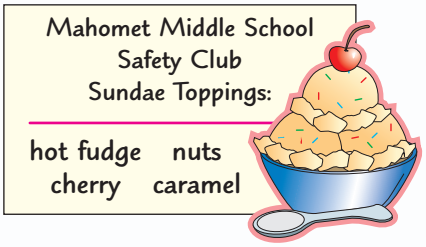
What You'll Learn
Find combinations.

Materials
- 5 index cards

Exploring Combinations

INVESTIGATE *Work with a partner.*

The student safety club at Mahomet Middle School is planning to sell sundaes with two different toppings at the summer carnival. The choices are shown at the right.



STEP 1 Write the name of the four sundae toppings on the index cards.

STEP 2 To make a sundae, select any pair of cards. Make a list of all the different combinations that are possible. Note that the order of the toppings is not important.

Cherry and Nuts

is considered the same as

Nuts and Cherry

Writing Math

Work with a partner.

1. How many different combinations are possible with two toppings?
2. How many different sundaes could be made with two toppings if the order of the toppings *was* important? What is this type of arrangement called?
3. **Write** a fifth topping on an index card. Now find all two-topping sundae combinations. How many are there? How many sundaes can be made with two toppings if the order of the toppings was important?
4. **Compare and contrast** the way you can find a permutation with the way you find a combination.
5. **Make a conjecture** about how to find a combination given a permutation.

9-5 Combinations

What You'll Learn
Find the number of combinations of a set of objects.

NEW Vocabulary
combination

LINK To Reading
Everyday Meaning of combination: the result of putting together objects, as in a combination of ingredients

WHEN am I ever going to use this?

BASKETBALL Coach Chávez wants to select co-captains for her basketball team. She will select two girls from the four oldest members on the team: Alita, Bailey, Charmaine, and Danielle.

1. Use the first letter of each name to list all of the permutations of co-captains. How many are there?
2. Cross out any arrangement that contains the same letters as another one in the list. How many are there now?
3. Explain the difference between the two lists above.

An arrangement, or listing, of objects in which order is *not* important is called a **combination**. For example, in the activity above, choosing Alita and Bailey is the same as choosing Bailey and Alita.

Permutations and combinations are related. You can find the number of combinations of objects by dividing the number of permutations of the entire set by the number of ways each smaller set can be arranged.

A permutation of 4 players, taken 2 at a time.

$$\frac{4 \cdot 3}{2!} = \frac{4 \cdot 3}{2 \cdot 1} = \frac{12}{2} = 6$$

There are 2! ways to arrange 2 players.

EXAMPLE 1 Find the Number of Combinations

FOOD **Paul's Pizza Parlor is offering a large two-topping pizza for $14.99. There are five toppings from which to choose. How many different two-topping pizzas are possible?**

Method 1 Make a list.

The five toppings are labeled pepperoni (p), sausage (s), onions (o), mushrooms (m), and green pepper (g).

p, s	p, m	p, o	p, g	s, m
s, o	s, g	m, o	m, g	o, g

Method 2 Use a permutation.

There are $5 \cdot 4$ permutations of two toppings chosen from five.

There are 2! ways to arrange the two toppings.

$$\frac{5 \cdot 4}{2!} = \frac{20}{2} = 10$$

So, there are 10 different two-topping pizzas.

EXAMPLE 2 Use a Combination to Solve a Problem

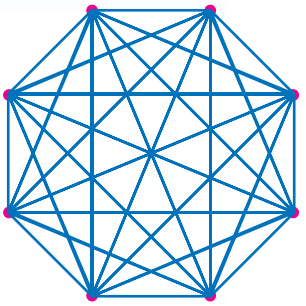
CHECKERS **A checkers tournament features each of the top 8 regional players playing every opponent one time. The 2 players with the best records will then play in a final round to determine the champion. How many matches will be played if there are no ties?**

Find the number of ways 2 players can be chosen from a group of 8.

There are $8 \cdot 7$ ways to choose 2 people. → $\frac{8 \cdot 7}{2!} = \frac{56}{2} = 28$
There are 2! ways to arrange 2 people. →

There are 28 matches plus 1 final match to determine the champion. So, there will be 29 matches played.

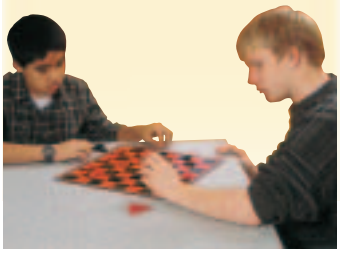
Check Make a diagram in which each person is represented by a point. Draw line segments between two points to represent the games. There are 28 line segments. Then add the final-round match to make a total of 29 matches.



REAL-LIFE MATH

CHECKERS Checkers is played on an 8-by-8 board in the U.S., England, Australia, and Ireland. People in eastern Europe play on a 10-by-10 board.

Source: www.about.com



Your Turn

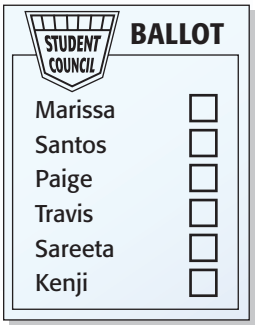
a. How many matches will be played if the top 16 players were invited to play?

The difference between permutations and combinations is that order is important in permutations, while order is *not* important in combinations.

EXAMPLES Identify Permutations and Combinations

Tell whether each situation represents a *permutation* or *combination*. Then solve the problem.

STUDENT GOVERNMENT **The six students listed at the right are members of Student Council. How many ways can you choose a president, vice president, and treasurer from this group?**



This is a permutation because the order of president, vice president, and treasurer is important. So, the number of ways you can choose the three officers is $6 \cdot 5 \cdot 4$, or 120 ways.

In how many ways can you choose a committee of three students from the six members in student council shown above?

This is a combination because the order of the students in the committee is not important.

There are $6 \cdot 5 \cdot 4$ ways to choose 3 people. → $\frac{6 \cdot 5 \cdot 4}{3!} = \frac{120}{6} = 20$
There are 3! ways to arrange 3 people. →

So, there are 20 ways to choose the committee.

Geoff Butler
CONTENTS

Skill and Concept Check

Writing Math
Exercises 1 & 3

1. **Explain** why a combination lock should be called a permutation lock.

2. **OPEN ENDED** Give an example of a permutation and combination.

3. **FIND THE ERROR** Allison and Francisca are calculating the number of ways that a 3-member committee can be chosen from a 7-member club. Who is correct? Explain.

Allison	Francisca
$7 \cdot 6 \cdot 5 = 210$ ways	$\frac{7 \cdot 6 \cdot 5}{3!} = \frac{210}{6} = 35$ ways

GUIDED PRACTICE

4. **VOLLEYBALL** Coach Malone has an 8-member volleyball team. He told his team that he would start six different players every game. How many games would it take to do this?

Tell whether each problem represents a *permutation* or *combination*. Then solve the problem.

5. How many ways can 10 students finish first, second, or third at the science fair?
6. How many ways can you pick 2 puppies from a litter of 7 puppies?

Practice and Applications

HOMEWORK HELP

For Exercises	See Examples
7–10	1, 2
11–16	3, 4

Extra Practice
See pages 586, 604.

7. **FOOD** The International Club is selling hot dogs at the Spring Carnival. Customers can select three toppings from among chili, onions, cheese, mustard, or relish. How many combinations of three-topping hot dogs are there?

CIVICS **For Exercises 8 and 9, use the information below and at the right.**
If five of the nine Justices on the 2002 United States Supreme Court agree on a decision, they can issue a majority opinion.

8. How many different combinations of five Supreme Court Justices are there?
9. Before they take the Bench each day, the Justices engage in the "Conference handshake." Each Justice shakes hands with each of the other eight. How many handshakes take place?

10. **SOCCER** There are 21 players trying out for 15 spots on the soccer team. How many ways does the coach have to create her team?

U.S. Supreme Court, 2002	
Chief Justice, William H. Rehnquist	
Stephan G. Breyer	Antonin Scalia
Ruth Bader Ginsburg	David H. Souter
Anthony M. Kennedy	John Paul Stevens
Sandra Day O'Connor	Clarence Thomas

Source: www.washingtonpost.com

Tell whether each problem represents a *permutation* or *combination*. Then solve the problem.

11. How many ways can you select four essay questions out of a total of 10 on the exam?
12. Six children remain in a game of musical chairs. If two chairs are removed, how many different groups of four students can remain?
13. How many ways can three flute players be seated in the first, second, or third seats in the orchestra?
14. In how many ways can four paintings be displayed from a collection of 15?
15. How many ways can seven students line up to buy concert tickets?
16. Given 12 Web sites, how many ways can you visit half of them?
17. **WRITE A PROBLEM** Write about a real-life situation that can be solved using a permutation and one that can be solved using a combination. Then solve both problems.
18. **CRITICAL THINKING** At a party, there were 105 handshakes. If each person shook hands exactly once with every other person, how many people were at the party?

Standardized Test Practice and Mixed Review

ISTEP+ Practice

19. **MULTIPLE CHOICE** In how many ways can three gymnasts from a team of 10 be chosen to compete in a meet?

    (A) 120 (B) 180 (C) 240 (D) 720

20. **SHORT RESPONSE** Find $\frac{(n+6)(n+5)(n+4)}{n!}$ if $n = 3$.
21. **SAILING** When Mr. Elms purchased his sailboat, it came with six different-colored flags to be used for sending signals. A specific signal depended on the order of the flags. How many different three-flag signals can he send? (Lesson 9-4)
22. **CARS** A certain brand of car has a choice of a 2.5, 3.1, or 4.0 liter engine; a radio with a cassette-player, CD-player, or CD-changer; cloth or leather seats; and silver, green, red, or white. How many different cars are possible? (Lesson 9-3)

Estimate. (Lesson 6-1)

23. $\frac{1}{10} + \frac{7}{8}$
24. $\frac{5}{12} - \frac{1}{9}$
25. $\frac{4}{9} \cdot 20$
26. $15\frac{7}{9} \div 3\frac{3}{5}$

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL Suppose you choose a card from a set of eight cards labeled 1–8. Write the probability of each event as a fraction in simplest form. (Lesson 9-1)

27. P(even number)
28. P(number greater than 6)
29. P(multiple of 8)
30. P(number less than 9)
31. P(prime number)
32. P(*not* 2 or 5)

9-6a Problem-Solving Strategy

A Preview of Lesson 9-6

Academic Standards
7.6.2, 7.7.1, 7.7.5, 7.7.12

What You'll Learn
Solve problems by acting them out.

Act It Out

I wonder if tossing a coin would be a good way to answer a 5-question true-false quiz?

I'm not so sure, Whitney. Let's do an experiment with a coin and **act it out**!

Explore

We know that there are five true-false questions on the quiz. We can carry out an experiment to test whether tossing a coin would be a good way to answer the questions and get a good grade.

Plan

Let's toss a coin five times. If the coin shows tails, we will answer T. If the coin shows heads, we will answer F. Let's do four trials.

Solve

Suppose the correct answers are T, F, F, T, F. Let's circle them.

Answers	T	F	F	T	F	Number Correct
Trial 1	(T)	T	(F)	F	T	2
Trial 2	F	(F)	T	(T)	(F)	3
Trial 3	(T)	(F)	T	F	T	2
Trial 4	F	(F)	T	F	(F)	2

The experiment produces 2 or 3 correct answers.

Examine

There is an equally likely chance that tossing a coin will produce a correct answer or a wrong answer for each question. Since the experiment produced about 2–3 correct answers on a 5-question quiz, it shows that tossing a coin to answer a true-false quiz is not the way to get a good grade.

Analyze the Strategy

1. **Explain** whether the results of the experiment would be the same if it were repeated.
2. **Explain** an advantage of using the act it out strategy to solve a problem.

Laura Sifferlin

Apply the Strategy

Solve. Use the act it out strategy.

3. **POP QUIZ** Determine whether using a spinner with four equal sections is a good way to answer a 5-question multiple-choice quiz. Each question has choices A, B, C, and D. Explain.

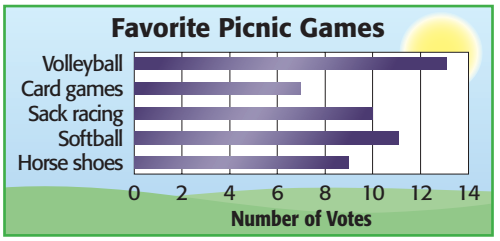
4. **PHOTOGRAPHS** Samuel is taking a picture of the Spanish Club's five officers. The club president will always stand on the left, and the vice president will always stand on the right. How many different ways can he arrange the officers for the picture?

Mixed Problem Solving

Solve. Use any strategy.

5. **MONEY MATTERS** Lola purchased a $35 book bag at a sale price of $27.50. What was the percent of decrease from the original price to the sales price?

6. **PICNIC** Examine the bar graph.



What is the probability that a student's favorite picnic game is sack racing?

7. **CROQUET** Sixteen teams are playing in a croquet tournament. If a team loses one game, it is eliminated. How many total games will be played in the tournament?

8. **EARTH SCIENCE** The Hawaiian mountain Mauna Kea is about 1.38×10^4 feet tall. Mt. Everest is about 29,000 feet. Which mountain is taller?

9. **BASEBALL** Each package of baseball cards contains a puzzle piece. If you collect all 6 different pieces, you win two tickets to a major league game. There is an equally-likely chance of getting a different puzzle piece each time. Explain how you would model this problem to find the answer.

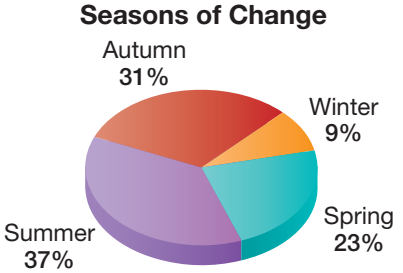
10. **PATTERNS** The pattern below is known as Pascal's Triangle. Find the pattern and complete the 6th and 7th rows.

1st Row				1					
2nd Row			1		1				
3rd Row		1		2		1			
4th Row		1		3		3		1	
5th Row	1		4		6		4		1

11. **SPACE SCIENCE** A 21-kilogram sample of rocks from the moon is composed of about 40% oxygen and about 19.2% silicon. How much more is the oxygen mass of the rocks than the silicon mass?

12. **TRANSPORTATION** A taxi charges $2.35 for the first 0.4 mile and $0.75 for each additional 0.4 mile. Find the cost of a 4-mile taxi ride.

13. **STANDARDIZED TEST PRACTICE** ISTEP+ Practice
In a middle school of 900 students, about how many prefer the season of autumn?



(A) 81 (B) 207 (C) 279 (D) 333

You will use the act it out strategy in the next lesson.

9-6 Theoretical and Experimental Probability

Academic Standards
7.6.5

What You'll Learn
Find and compare experimental and theoretical probabilities.

NEW Vocabulary
- **experimental probability**
- **theoretical probability**

REVIEW Vocabulary
- **sample space:** a set of all possible outcomes **(Lesson 9-2)**

HANDS-ON Mini Lab

Materials
- 2 number cubes

Work with a partner.

STEP 1 Roll two number cubes 36 times. Record the number of times the sum of the number cubes is 7.

STEP 2 Use an addition table from 1 to 6 to help you find the expected number of times the sum of 7 should come up after rolling two number cubes 36 times. The top row represents one number cube, and the left column represents the other number cube. The table above has been started for you.

+	1	2	3
1	2	3	4
2	3	4	5

1. How many times did you roll a sum of 7? What is the probability of rolling a sum of 7?
2. How does your result compare to the results of other groups? Explain.
3. What is the expected probability of rolling a sum of 7?
4. How does your result compare to the expected probability of rolling a sum of 7? Explain any differences.

In activity above, you found the experimental probability of rolling a sum of 7 on two number cubes. **Experimental probability** is found using frequencies obtained in an experiment or game.

The expected probability of an event occurring is called **theoretical probability**. This is the probability that you have been using since Lesson 9-1. The theoretical probability of rolling a sum of 7 on two number cubes is $\frac{6}{36}$, or $\frac{1}{6}$.

EXAMPLE 1 Experimental Probability

Two number cubes are rolled seventy-five times and a sum of 9 is rolled ten times.

What is the experimental probability of rolling a sum of 9?

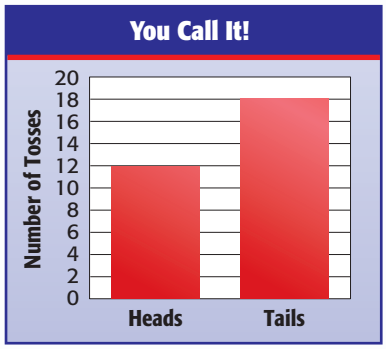
$$P(9) = \frac{\text{number of times a sum of 9 occurs}}{\text{number of possible outcomes}}$$

$$= \frac{10}{75} \text{ or } \frac{2}{15}$$

The experimental probability of rolling a sum of 9 is $\frac{2}{15}$.

EXAMPLES Experimental and Theoretical Probability

2 **The graph shows the results of an experiment in which a coin was tossed thirty times. Find the experimental probability of tossing tails for this experiment.**



$$P(\text{tails}) = \frac{\text{number of times tails occurs}}{\text{number of possible outcomes}}$$
$$= \frac{18}{30} \text{ or } \frac{3}{5}$$

The experimental probability of tossing tails is $\frac{3}{5}$.

3 **Compare the experimental probability you found in Example 2 to its theoretical probability.**

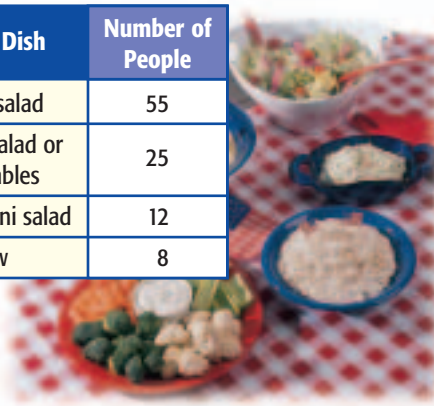
The theoretical probability of tossing tails on a coin is $\frac{1}{2}$. So, the experimental probability is close to the theoretical probability.

Experimental probability can also be based on past performances and be used to make predictions on future events.

EXAMPLES Predict Future Events

4 **FOOD** **In a survey, 100 people were asked to name their favorite Independence Day side dishes. What is the experimental probability of macaroni salad being someone's favorite dish?**

Side Dish	Number of People
potato salad	55
green salad or vegetables	25
macaroni salad	12
coleslaw	8



There were 100 people surveyed and 12 chose macaroni salad. So, the experimental probability is $\frac{12}{100}$, or $\frac{3}{25}$.

5 **Suppose 250 people attend the city's Independence Day barbecue. How many can be expected to choose macaroni salad as their favorite side dish?**

$\frac{3}{25} = \frac{x}{250}$ Write a proportion.

$3 \cdot 250 = 25x$ Find the cross products.

$30 = x$ About 30 will choose macaroni salad.

Your Turn

a. What is the experimental probability of potato salad being someone's favorite dish?

b. About how many people can be expected to choose potato salad as their favorite dish if 250 attend the barbecue?

REAL-LIFE CAREERS

How Does a Chef Use Math?

In addition to cooking, chefs order enough food and supplies based on customers' preferences and recent sales.

Online Research

For more information about a career as a chef, visit: msmath2.net/careers

CONTENTS

Skill and Concept Check

Writing Math — Exercises 1 & 2

1. **Compare and contrast** experimental probability and theoretical probability.

2. **OPEN ENDED** Give an example of a situation that has a theoretical probability of $\frac{1}{10}$.

GUIDED PRACTICE

For Exercises 3 and 4, a coin is tossed 50 times, and it lands heads 28 times.

3. Find the experimental probability of the coin landing heads.
4. Find the theoretical probability of the coin landing heads.

5. **TRAFFIC** Between 3:00 P.M. and 4:00 P.M., 11 sedans, 15 trucks, and 6 sports cars drove through an intersection. Based on this information, what is the probability that a vehicle that drives through the intersection is a sports car?

Practice and Applications

HOMEWORK HELP

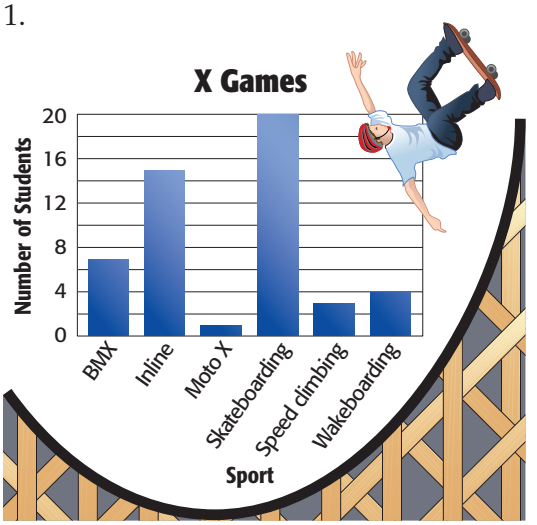
For Exercises	See Examples
6–9	1–3
10, 15	4
11–14, 16–17	5

Extra Practice
See pages 586, 604.

For Exercises 6–9, a number cube is tossed 20 times and lands on 1 two times and on 5 four times.

6. Find the experimental probability of landing on 5.
7. Find the theoretical probability of *not* landing on 5.
8. Find the theoretical probability of landing on 1.
9. Find the experimental probability of *not* landing on 1.

X GAMES **For Exercises 10–12, use the graph of a survey of 50 students asked to name their favorite X Game sport.**



10. What is the probability of inline being someone's favorite sport?
11. Suppose 500 people attend the X Games. How many can be expected to choose inline as their favorite sport?
12. Suppose 500 people attend the X Games. How many can be expected to choose speed climbing as their favorite sport?

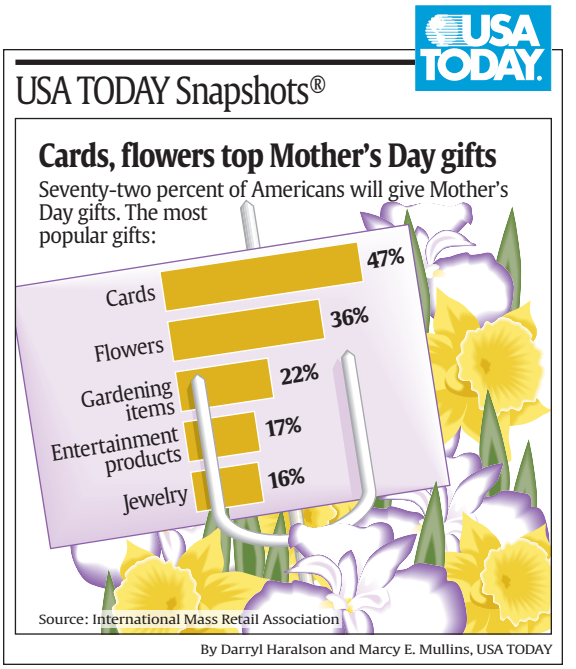
13. **REFRESHMENTS** In a survey taken at the beach, 47 people preferred cola, 28 preferred root beer, and 25 preferred ginger ale. If the manager of the Beach Hut is going to buy 50 cases of soda for the next day, about how many cases should be root beer?

14. **SPINNERS** A spinner marked with three sections A, B, and C was spun 100 times. The results are shown in the table. Make a drawing of the spinner based on its experimental probabilities.

Section	Frequency
A	24
B	50
C	26

GIFTS **For Exercises 15–17, use the results of the survey at the right.**

15. What is the probability that a mother will receive a gift of flowers? Write the probability as a fraction.
16. Out of 750 mothers that receive gifts, how many would you expect to receive flowers?
17. Out of 750 mothers that receive gifts, how many would you expect to receive jewelry?



18. **CRITICAL THINKING** *Hot numbers* in a lottery are numbers that keep coming up. *Cold numbers* are numbers that have not "hit" in awhile. Some people select hot numbers or cold numbers when buying a ticket. Using probability, explain whether this makes sense.

EXTENDING THE LESSON

The *odds* of an event occurring is the ratio that compares the number of ways an event can occur (success) to the number of ways it cannot occur (failure).

Example Find the odds of rolling a number less than 5 on a number cube.

ways to occur: 4 — ways *not* to occur: 2

sample space: 1, 2, 3, 4, 5, 6 → 4 : 2, or 2 : 1

Find the odds of each outcome if a number cube is rolled.

19. 6
20. odd number
21. number greater than 4

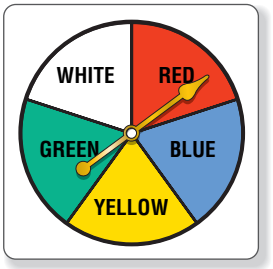
Standardized Test Practice and Mixed Review

ISTEP+ Practice

22. **MULTIPLE CHOICE** Myron has four dimes in his wallet with dates 1998, 1995, 2002, and 2000. If he randomly picks one dime from his wallet, what is the probability that it will have a date in the 1990s?

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

23. **GRID IN** Rachel spins the spinner at the right 50 times, and it lands on red 15 times. What is the theoretical probability of spinning red?



24. **BASEBALL** How many ways can a baseball coach select four starting pitchers from a pitching staff of eight? (Lesson 9-5)
25. **POLE VAULT** A pole vault competition has 10 people in it. In how many ways can first-, second- and third-place ribbons be awarded? (Lesson 9-4)

GETTING READY FOR THE NEXT LESSON

PREREQUISITE SKILL **Three coins are tossed. Find each probability.** (Lesson 9-1)

26. P(3 tails)
27. P(2 heads)
28. P(at least 2 heads)
29. P(at least 1 tail)

9-6b HANDS-ON LAB

A Follow-Up of Lesson 9-6

Academic Standards
7.6.5

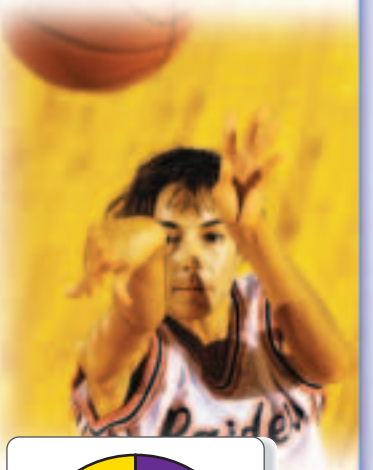
What You'll Learn

Investigate experimental probability.

Materials

- paper
- scissors
- spinner arrows

Experimental Probability



INVESTIGATE *Work with a partner.*

Shannon is on the basketball team. She makes 75% of her free throws. In this lab, you will model, or *simulate* Shannon shooting free throws with a spinner and investigate the experimental probabilities of her being able to make a free throw.

- Create and cut out the spinner at the right.

- Spin the spinner 25 times to model Shannon shooting 25 free throws. Record your results in the table below. If the spinner lands on a line, spin again.

Outcome	Tally	Total
Made Free Throw		
Missed Free Throw		

- Combine your data with four other groups so you have 100 data points.

Writing Math

Work with a partner.

1. **Explain** why you combined your data with four other groups.
2. From your table, what is the experimental probability of making a free throw?
3. How does the experimental probability compare with Shannon's past performance?
4. What is the experimental probability of making two free throws in a row? **Explain** how you found your answer.
5. Suppose Shannon did shoot 100 free throws in a row. What factors would influence her making *or* not making 75 free throws? **Explain** your reasoning.
6. **Describe** the size of both sections of the spinner.
7. **Describe** another way to simulate Shannon shooting free throws.
8. **Draw** a spinner that simulates a free-throw shooter making 60% of her free throws.

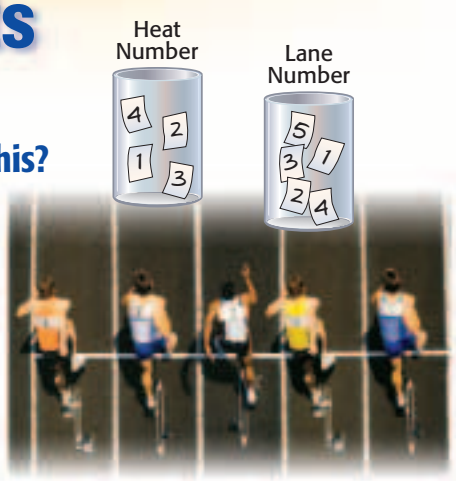
9-7 Independent and Dependent Events

Academic Standards
7.6.6

What You'll Learn
Find the probability of independent and dependent events.

NEW Vocabulary
compound event
independent event
dependent event

WHEN am I ever going to use this?



TRACK AND FIELD The 100-meter dash features 20 runners competing in a preliminary round of 4 heats. The winner of each heat advances to the final race. Before the race, each runner chooses a number from jar 1 to determine the heat in which he runs and a number from jar 2 to determine one of five lanes he occupies. Omar is the first runner to choose from the jars.

1. What is the probability of Omar being in the second heat?
2. What is the probability of Omar being in lane 3?
3. Multiply your answers in Exercises 1 and 2 above. What does this number mean? Explain.

In the Mini Lab, choosing the heat and the lane is a compound event. A **compound event** consists of two or more simple events. Since choosing the heat number does not affect choosing the lane number, both events are called **independent events**.

Key Concept — **Probability of Two Independent Events**

Words The probability of two independent events can be found by multiplying the probability of the first event by the probability of the second event.

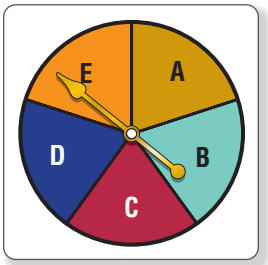
Symbols $P(A \text{ and } B) = P(A) \cdot P(B)$

READING Math

Probability Notation
$P(A \text{ and } B)$ is read *the probability of A followed by B.*

EXAMPLE 1 Independent Events

A number cube is rolled, and the spinner at the right is spun. Find the probability of rolling a 2 and spinning a vowel.



$P(2) = \frac{1}{6}$ $\quad P(\text{vowel}) = \frac{2}{5}$

$P(2 \text{ and vowel}) = \frac{1}{\cancel{6}_{3}} \cdot \frac{\cancel{2}^{1}}{5}$ or $\frac{1}{15}$

So, the probability of rolling a 2 and spinning a vowel is $\frac{1}{15}$.

Check You can make a tree diagram to check your answer.

CONTENTS

If the outcome of one event affects the outcome of a second event, the events are called **dependent events**.

EXAMPLE 2 Dependent Events

SNACK BARS **A box contains 2 oatmeal, 3 strawberry, and 6 cinnamon snack bars. Ruby reaches in the box and randomly takes two snack bars, one after the other. Find the probability that she will choose a cinnamon bar and then a strawberry bar.**

$P(\text{cinnamon}) = \frac{6}{11}$ ← 11 snack bars, 6 are cinnamon

$P(\text{strawberry}) = \frac{3}{10}$ ← 10 snack bars after 1 cinnamon snack bar has been removed, 3 are strawberry

$$P(\text{cinnamon, then strawberry}) = \frac{\cancel{6}^{3}}{11} \cdot \frac{3}{\cancel{10}_{5}} \text{ or } \frac{9}{55}$$

So, the probability that Ruby will choose a cinnamon snack bar and then a strawberry snack bar is $\frac{9}{55}$, or about 16%.

STUDY TIP

Dependent Events
The probability of *B* following *A* is the same as the probability of *A* following *B*.

Key Concept **Probability of Two Dependent Events**

Words The probability of two dependent events is the probability of the first event times the probability that the second event occurs after the first.

Symbols $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$

Skill and Concept Check

Writing Math
Exercises 1 & 3

1. **Explain** the difference between independent and dependent events.
2. **OPEN ENDED** Describe two dependent events.
3. **FIND THE ERROR** Kimi and Shane are finding the probability of spinning an even number on two consecutive spins. Who is correct? Explain.

Kimi
$\frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20} = \frac{1}{10}$

Shane
$\frac{2}{5} \cdot \frac{2}{5} = \frac{4}{25}$

GUIDED PRACTICE

4. A coin is tossed, and a number cube is rolled. What is the probability of tossing heads and rolling a 3 or a 5?
5. A bag contains 5 red apples and 3 yellow apples. What is the probability of picking 2 red apples without the first being replaced?

Practice and Applications

HOMEWORK HELP

For Exercises	See Examples
6–7, 17–19	1
8–9, 11–14	2

Extra Practice
See pages 586, 604.

6. A red and a blue number cube are rolled. Find the probability that an odd number is rolled on the red cube and a number greater than 1 is rolled on the blue cube.

7. Find the probability of heads on three consecutive tosses of a coin.

8. A cooler is filled with 12 colas and 9 diet colas. If Victor randomly chooses two without replacing the first, what is the probability that he will choose a cola and then a diet cola?

9. A deck of 30 cards is made up of the numbers 1–10 in three colors: red, purple, and green. Two cards are selected without either being replaced. Find the probability of choosing a purple 5 and then a red or green card.

10. Draw a Venn diagram to show the probability of two independent events A and B.

11. **CIVICS** In the 108th Congress, Tennessee had 4 Republicans and 5 Democrats serving in the House of Representatives. If a subcommittee of 2 representatives was formed to study Internet usage among middle school students, what is the probability that both would be Republicans?

CARDS **For Exercises 12–14, use the information below.**
A standard deck of playing cards contains 52 cards in four suits of 13 cards each. Two suits are red and two suits are black. Two cards are chosen from the deck one after another. Find each probability.

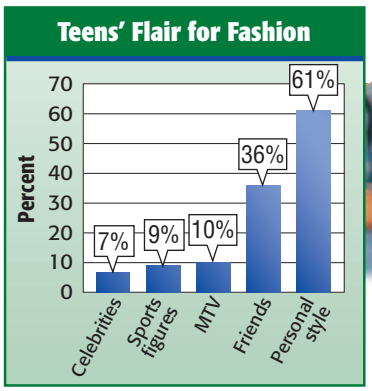
12. P(2 hearts)
13. P(red, black)
14. P(Ace, King)

Determine whether each event is *independent* or *dependent*.

15. choosing a student in the 7th grade and a student from the 8th grade
16. choosing a pair of shoes to try on, then choosing a smaller pair to try on

FASHION **For Exercises 17 and 18, use the graphic at the right. Write your answers as decimals to the nearest hundredth.**

17. What is the probability that a teen chosen at random said that friends and sports figures influenced her fashion choices?
18. What is the probability that a teen chosen at random said that personal style and celebrities influenced his fashion choices?

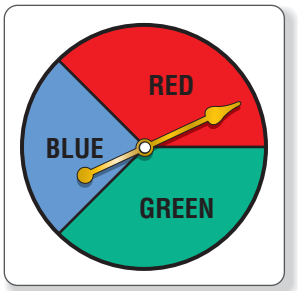


Source: American Express Retail Index

19. **TRAFFIC SIGNALS** There are three consecutive traffic signals on a street that operate independently of each other. The first is green 45% of the time, the second is green 60% of the time, and the third is green 50% of the time. What is the probability of a person driving down the street making all three green lights? Write as a percent to the nearest tenth.

CONTENTS

CRITICAL THINKING **For Exercises 20 and 21, use the spinner.**



20. Make a tree diagram of all the possible outcomes of three successive spins of the spinner shown. How many paths in the tree diagram represent two red spins and one blue?

21. Suppose the spinner is designed so that for each spin there is a 40% probability of spinning red and a 20% chance of spinning blue. What is the probability of spinning two reds and one blue?

EXTENDING THE LESSON Events that cannot happen at the same time are *mutually exclusive.* The two events are connected by the word *or* and the probability is found by *adding* their individual probabilities.

Example Suppose a number cube is rolled. Find P(even or 5).

$$P(\text{even or } 5) = \frac{3}{6} + \frac{1}{6} = \frac{4}{6} \text{ or } \frac{2}{3}$$

Find each probability as a fraction in simplest form.

22. P(1 or 3)

23. P(5 or multiple of 3)

24. P(6 or less than 5)

Standardized Test Practice and Mixed Review

ISTEP+ Practice

25. **MULTIPLE CHOICE** You guess the answers on a two-question multiple-choice quiz with five choices: a, b, c, d, and e. What is the probability that you will get both correct?

(A) $\frac{1}{50}$ (B) $\frac{1}{25}$ (C) $\frac{2}{25}$ (D) $\frac{1}{5}$

26. **GRID IN** Find the probability of rolling an even number on a number cube, tossing heads on a coin, and choosing an ace in a standard deck of cards.

27. **PROBABILITY** Paz performed a probability experiment by spinning a spinner 20 times. The results are shown in the table. If the spinner is divided into four equal sections, how many sections would you expect to be blue? (Lesson 9-6)

Color	Frequency
red	𝍸
green	𝍸 𝍸
blue	𝍸

28. **LOTTERY** Balls numbered 1 to 51 are dropped into a machine and mixed together. Six balls are then selected in any order to make up the winning numbers. How many different ways can the six numbers be chosen? (Lesson 9-5)

Evaluate each expression if $a = 6$, $b = -4$, and $c = -3$. (Lesson 3-6)

29. $9c$

30. $-8a$

31. $2bc$

32. $5b^2$

WebQuest Interdisciplinary Project

Step Right Up and Win a Prize

It's time to complete your project. Use the information and data you have gathered about carnival games to prepare a Web page or poster. Be sure to include a scale drawing of the game you design with your project.

msmath2.net/webquest

CHAPTER 9

Study Guide and Review

Vocabulary and Concept Check

combination (p. 387)
complementary events (p. 371)
compound event (p. 398)
dependent events (p. 399)
experimental probability (p. 393)
factorial (p. 381)
fair game (p. 374)
Fundamental Counting Principle (p. 378)
independent events (p. 398)
outcome (p. 370)
permutation (p. 381)
probability (p. 370)
random (p. 370)
sample space (p. 374)
simple event (p. 370)
theoretical probability (p. 393)
tree diagram (p. 374)

Choose the correct term to complete the sentence.

1. The set of all possible outcomes for an experiment is called the (sample space, probability).
2. The Fundamental Counting Principle counts the number of possible outcomes using the operation of (addition, multiplication).
3. The ratio of the number of times an event occurs to the number of trials completed is called the (theoretical, experimental) probability.
4. When the outcome of one event influences the outcome of a second event, the events are called (independent, dependent).
5. A (permutation, combination) is a listing of objects in which order is important.
6. A (complementary, compound) event consists of two or more simple events.
7. A (simple, random) event occurs by chance.
8. The probability of an event is the (product, ratio) of the number of ways an event can occur to the number of possible outcomes.
9. When using a combination, the order of the arrangement (is, is not) important.
10. The expression $n!$ is the (sum, product) of all counting numbers beginning with n and counting backward to 1.

Lesson-by-Lesson Exercises and Examples

9-1 Simple Events (pp. 370–373)

A bag contains 6 red, 3 pink, and 3 white bows. Suppose you draw a bow at random. Find the probability of each event. Write each fraction in simplest form.

11. P(red)
12. P(pink)
13. P(white)
14. P(red or white)
15. P(pink or white)
16. P(*not* white)

Example 1 **What is the probability of rolling an odd number on a number cube?**

$$P(\text{odd}) = \frac{\text{odd numbers possible}}{\text{total numbers possible}}$$

$= \frac{3}{6}$ Three numbers are odd: 1, 3, and 5.

$= \frac{1}{2}$ Simplify.

Therefore, $P(\text{odd}) = \frac{1}{2}$.

msmath2.net/vocabulary_review

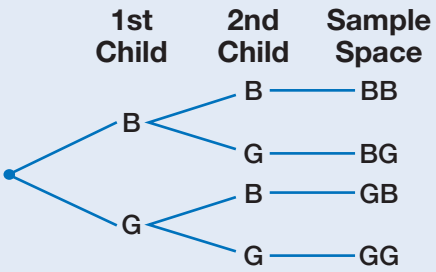
CONTENTS

9-2 Tree Diagrams (pp. 374–377)

For each situation, make a tree diagram to show the sample space. Then give the total number of outcomes.

17. rolling a number cube and then tossing a coin
18. choosing a red, blue, or white shirt with either black or gray lettering
19. tossing a coin and choosing a card from seven cards numbered from 1 to 7
20. choosing from white, wheat, or rye bread and turkey, ham, or salami to make a sandwich

Example 2 **Make a tree diagram to describe the possible outcomes for a family with two children.**



If a family has two children, there are 4 possible outcomes.

9-3 The Fundamental Counting Principle (pp. 378–380)

Use the Fundamental Counting Principle to find the total number of outcomes in each situation.

21. rolling two number cubes
22. selecting a car from 3 styles, 3 interior colors, and 3 exterior colors
23. making an ice cream sundae selecting from 5 flavors of ice cream and 4 different toppings
24. **SHOPPING** One catalog offers a jogging suit in two colors, gray and black. It comes in sizes S, M, L, XL, and XXL. How many possible jogging suits can be ordered?

Example 3 **Use the Fundamental Counting Principle to find the total number of outcomes for a family that has three children.**

There are 2 possible outcomes each time a family has a child, a boy or a girl. If a family has 3 children, there are $2 \cdot 2 \cdot 2$, or 8 outcomes.

9-4 Permutations (pp. 381–383)

Find the value of each expression.

25. $4!$
26. $9!$
27. $8 \cdot 7 \cdot 6 \cdot 5$
28. $3! \cdot 4!$
29. $5! \cdot 3!$
30. $7! \cdot 4!$
31. **BASKETBALL** In how many ways can five basketball players be placed in three positions?

Example 4 **In how many ways can a director and assistant director be chosen from among 4 candidates?**

There are 4 choices for director and then 3 choices remain for assistant director. So, there are $4 \cdot 3$, or 12, ways the director and assistant director can be chosen.

9-5 Combinations (pp. 387–390)

32. How many three-topping pizzas are possible given eight different toppings?
33. How many groups of five people are there from a committee of nine?
34. How many groups of three kittens are possible from a litter of nine?
35. In how many ways can Rondell select two board games from the ten that his family has?

Example 5 **In how many ways can you choose two items from a menu with four items?**

There are $4 \cdot 3$ permutations of two menu items chosen from four.

There are 2! ways to arrange the two menu items.

$\frac{4 \cdot 3}{2!} = \frac{12}{2}$ or 6

So, there are 6 ways to choose two items.

9-6 Theoretical and Experimental Probability (pp. 393–396)

The results of spinning a spinner labeled A–E fifty times are given. Find the experimental probability of each event.

Letter	Frequency
A	8
B	17
C	9
D	6
E	10

36. P(A) 37. P(D) 38. P(E)
39. If the spinner is equally likely to land on each section, what is the theoretical probability of landing on B?

Example 6 **A coin is tossed 75 times, and it lands on tails 55 times. What is the experimental probability of the coin landing on heads?**

The coin landed on heads 20 times.

$P(\text{heads}) = \frac{\text{number of times heads occurs}}{\text{number of possible outcomes}}$

$= \frac{20}{75}$ or $\frac{4}{15}$

So, the experimental probability of the coin landing on heads is $\frac{4}{15}$.

9-7 Independent and Dependent Events (pp. 398–401)

For Exercises 40–43, a bag contains 6 green, 8 white, and 2 blue counters. Two counters are randomly drawn. Find each probability if the first counter is replaced before the second counter is drawn.

40. P(green, blue) 41. P(2 white)

Find each probability if the first counter is *not* replaced before the second counter is drawn.

42. P(2 green) 43. P(blue, white)

Example 7 **A box contains 12 solid, 14 striped, and 10 spotted marbles. Suppose you reach in and grab two marbles. Find the probability of choosing a striped marble, replacing it, and then choosing a spotted marble.**

$P(\text{striped}) = \frac{14}{36}$

$P(\text{spotted, after replacing the striped}) = \frac{10}{36}$

$\frac{14}{36} \cdot \frac{10}{36} = \frac{140}{1{,}296}$ or $\frac{35}{324}$

So, the probability of choosing a striped marble, replacing it, and then choosing a spotted marble is $\frac{35}{324}$, or about 11%.

CHAPTER 9

Practice Test

Vocabulary and Concepts

1. **Explain** the purpose of a tree diagram.
2. **Define** *combination*.

Skills and Applications

A spinner with sections labeled 1–8 has an equal chance of landing on each number. Find each probability.

3. P(odd number)
4. P(number greater than 1)
5. P(1 or 7)

For each situation, make a tree diagram to show the sample space. Then give the total number of outcomes.

6. tossing a coin three times
7. choosing a letter from the word MATH and then a digit from the number 123

Use the Fundamental Counting Principle to find the total number of outcomes in each situation.

8. choosing a 3-digit security code
9. rolling four number cubes

Find the value of each expression.

10. $8!$
11. $10 \cdot 9 \cdot 8 \cdot 7$
12. $5! \cdot 4!$

13. **PARADES** If there are 50 floats in a parade, how many ways can a first place and a second place trophy be awarded?

14. **SURVEY** Two hundred fifty teenagers were asked what type of pet they owned. The results of the survey are in the table. What is the experimental probability that a teenager owns a pet? Write as a percent.

Pet	Number of Teenage Pet Owners
fish	26
cat	65
dog	86
bird	20
other	38
no pet	15

15. **YOGURT** A brand of yogurt has 15 different flavors. In how many ways can you choose three flavors?

ISTEP+ Practice — Standardized Test Practice

16. **GRID IN** A variety bag of popcorn contains 10 pieces of regular popcorn, 6 pieces of cheese popcorn, and 4 pieces of caramel popcorn. Katie pulls out one piece of popcorn and eats it. Then she pulls out a second piece and eats it. What is the probability that her first piece was caramel and her second piece was regular?

CHAPTER 9

Standardized Test Practice

ISTEP+ Practice

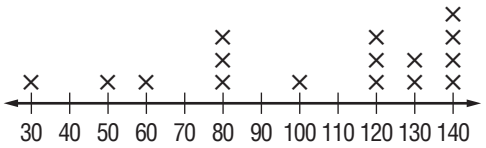
PART 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. The ornithologist said there is a 0.6754 chance of the bird returning next year to its nest in a tree in the Glowinski family's backyard. What is the value of the 5 in this number? (Prerequisite Skill, p. 555)

   (A) five tenths
   (B) five hundredths
   (C) five thousandths
   (D) five ten thousandths

2. The line plot below shows the weight in grams of a single serving of different brands of cookies. Between which weights is there a cluster of data? (Lesson 2-3)



   (F) between 30 and 50
   (G) between 60 and 80
   (H) between 90 and 110
   (I) between 120 and 140

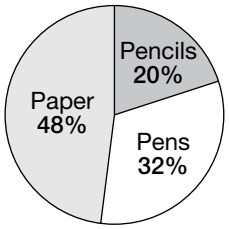
3. Park rangers measured the depth of a nearby river. It is 12 inches shallower than it was 4 years ago. Which formula best expresses the yearly change in the river? (Lesson 3-7)

   (A) $-12 \div 4$ (B) 12×4
   (C) -12×4 (D) $-12 \div -4$

4. Which of the following is the greatest common factor of 84 and 49? (Lesson 5-2)

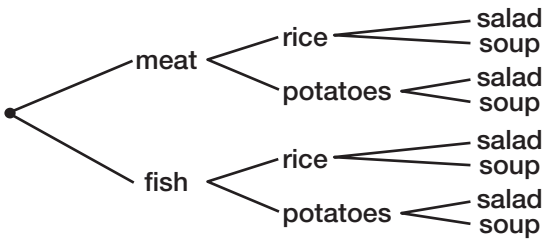
   (F) 2 (G) 6
   (H) 7 (I) 12

5. If \$792 was made in the sale of pens, what was the total amount of all sales to the nearest dollar? (Lesson 8-2)



   (A) \$2,534 (B) \$2,475
   (C) \$253 (D) \$248

6. The tree diagram shows the number of ways a customer can order lunch in a diner. If all choices are equally likely, what is the probability that a customer will order meat, rice, and salad? (Lesson 9-2)



   (F) $\frac{1}{16}$ (G) $\frac{1}{8}$ (H) $\frac{1}{4}$ (I) $\frac{3}{5}$

7. Fernando has to wash carrots, celery, onions, and potatoes for dinner. Assuming that he washes each vegetable only once, in how many different orders can Fernando wash all of the vegetables? (Lesson 9-4)

   (A) 4 (B) 16 (C) 20 (D) 24

8. What is the theoretical probability of choosing a vowel from the word MATHEMATICS? (Lesson 9-6)

   (F) $\frac{4}{11}$ (G) $\frac{4}{9}$ (H) $\frac{4}{7}$ (I) $\frac{7}{11}$

TEST-TAKING TIP

Question 7 Find your own answer before looking at the answer choices. Doing so keeps you from being tempted by wrong answer choices that look correct, but are still wrong.

ISTEP+ Practice

PART 2 Short Response/Grid In

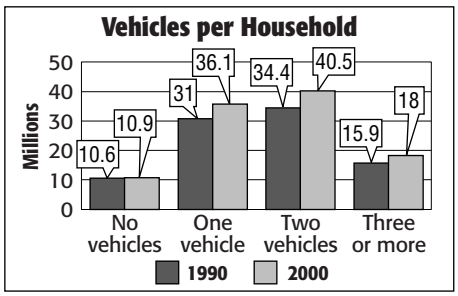
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. Ebony shot −4, −2, 1, and −3 in four rounds of golf. The final score is the sum of the scores. What was her final score? (Lesson 3-4)

10. How much greater is the area of a professional basketball court than a college basketball court? (Lesson 6-8)

Basketball Court	Length (ft)	Width (ft)
college	84	50
professional	94	50

11. What is the percent of increase in two-vehicle households from 1990 to 2000? Write as a percent to the nearest tenth. (Lesson 8-4)



Source: Census Bureau

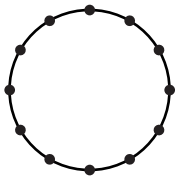
12. If two number cubes are rolled, what is the probability that their sum is 5 or 9? (Lesson 9-1)

13. Suppose you want to spend some time on the water. In how many ways can you choose a type of watercraft, the length of time, and the river? (Lesson 9-3)

Type of Watercraft	Time (hours)	River
paddleboat	1	Hamlin
canoe	2	Dustin
kayak	3	
water tricycle		

14. In how many ways can the Geiger family sit in a car if the two parents are in the front seat and the three children are in the back seat? (Lesson 9-4)

15. Twelve points are drawn on a circle. How many different line segments can be drawn between any two points? (Lesson 9-5)



For Questions 16 and 17, use the information below and at the right.

A survey asked 400 students what type of materials they prefer to read. The table shows the results. (Lesson 9-6)

Type of Reading Materials	Number of Students
book	225
newspaper	35
magazine	140

16. What is the probability that a student prefers to read a magazine?

17. Suppose 1,000 students attend Franklin Middle School. About how many would prefer to read a book?

ISTEP+ Practice

PART 3 Extended Response

Record your answers on a sheet of paper. Show your work.

18. Paula has five hobbies: jogging, painting, stamp collecting, quilting, and listening to music. She can enjoy her hobbies on either Friday night or Saturday night.
    a. Make a tree diagram that shows all of her possible outcomes. (Lesson 9-2)
    b. What is the probability that Paula will paint or jog on Friday night? (Lesson 9-2)
    c. Paula will not do the same hobby on both nights. What is the probability that Paula will quilt on Friday night and listen to music on Saturday night? (Lesson 9-7)